Tutorial on Fast Marching Method

Application to Trajectory Planning for Autonomous Underwater Vehicles

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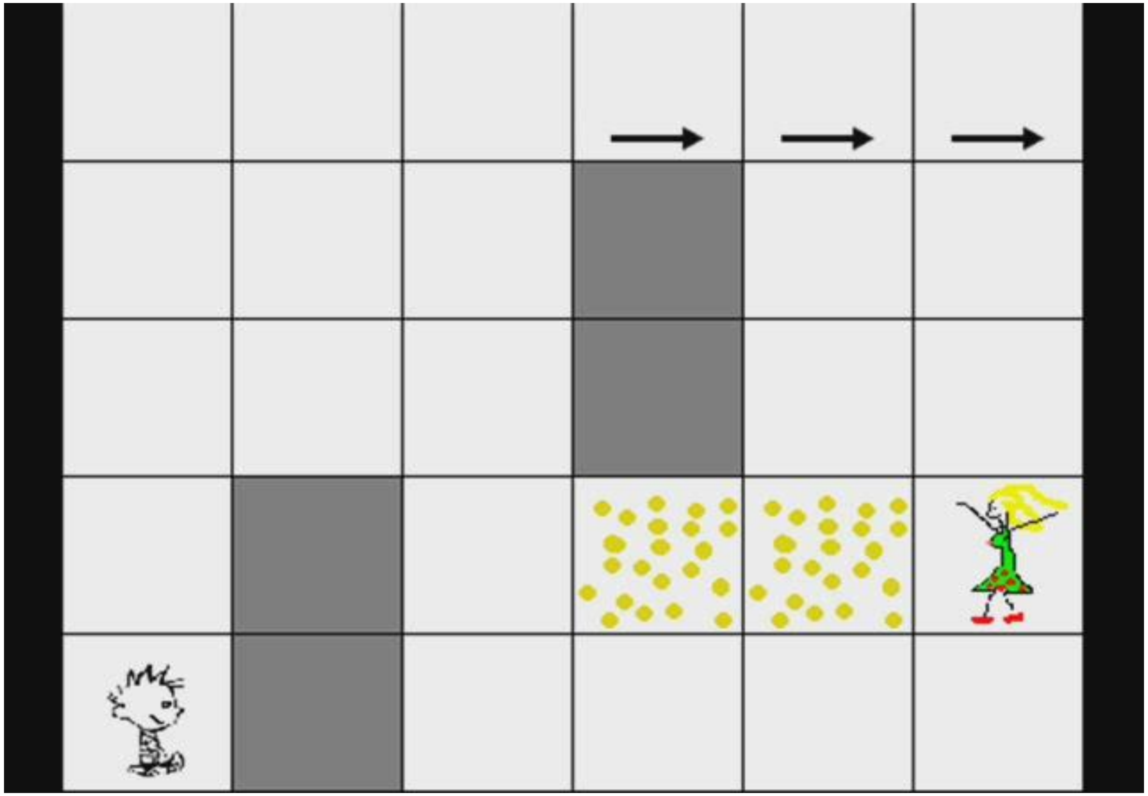
http://www.eece.hw.ac.uk/research/oceans/

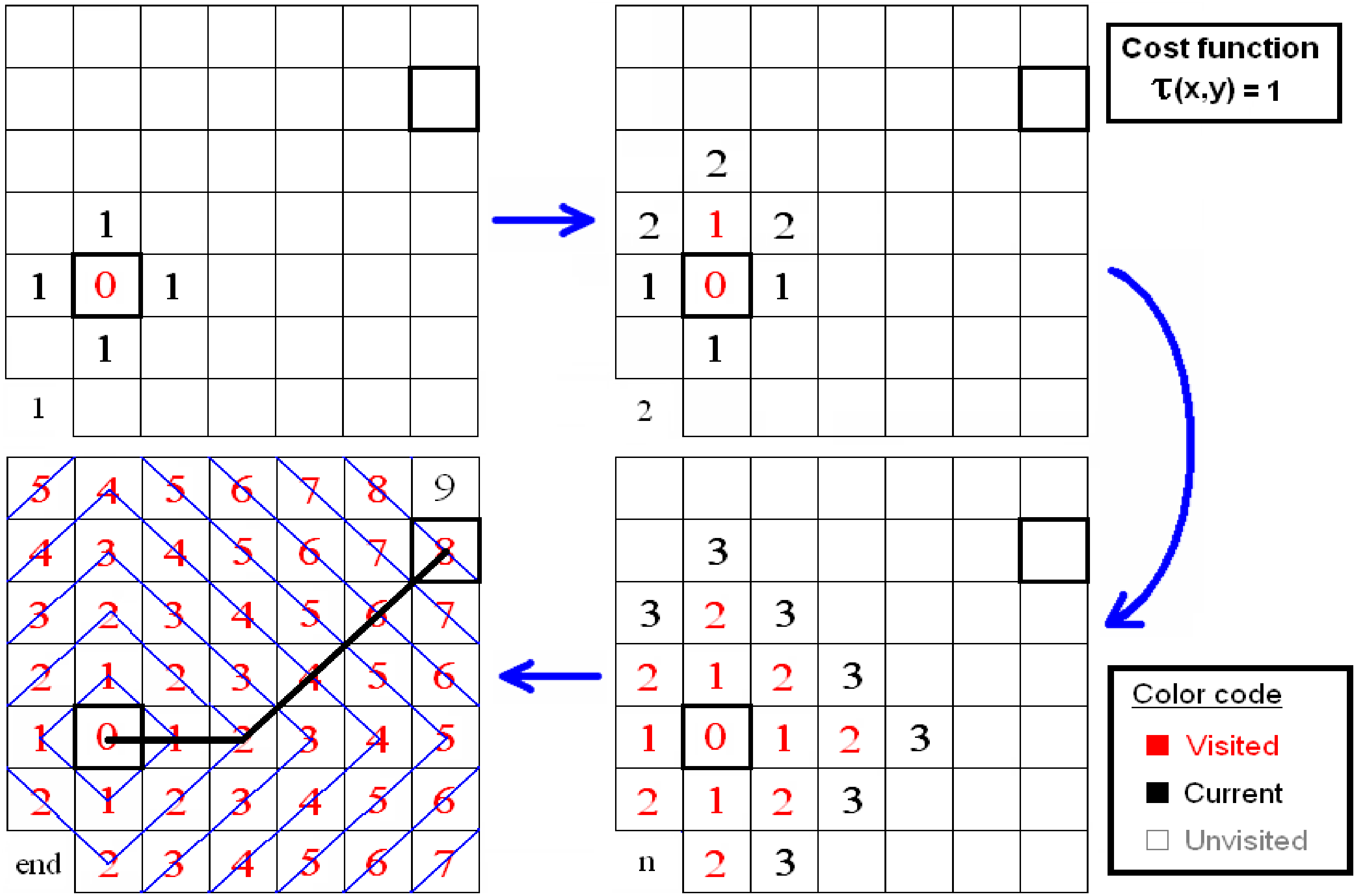
Presentation overview

Costs

- Obstacles: 11
- Rocky ground: 2
- Free space: 1
- Wind: 0.5



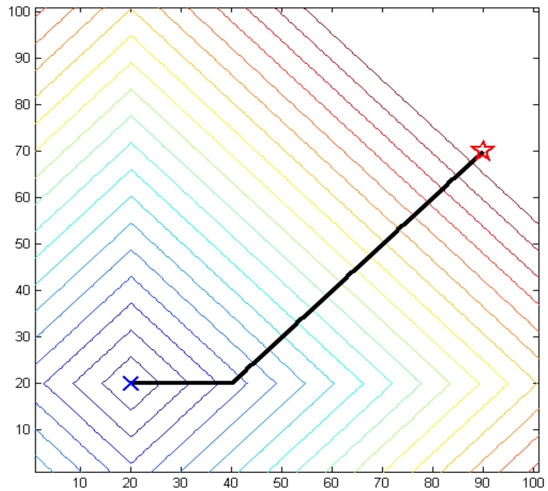
Grid-search algorithms



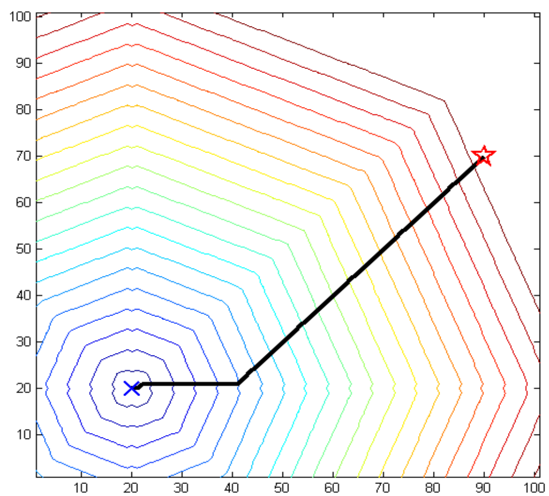
4-connexity Breadth-First algorithm without obstacle
(cost function = constant = 1)

Some demos (priority queue) …

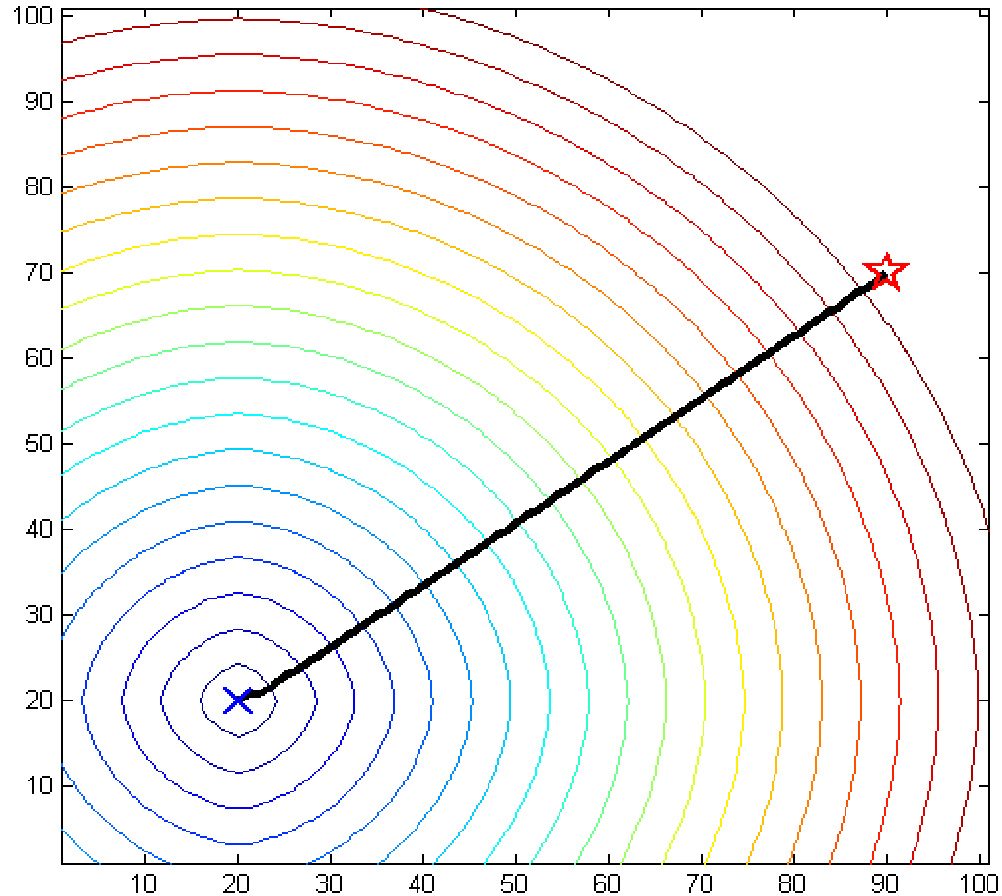
Towards Fast Marching algorithm…



4-connexity Breadth-First



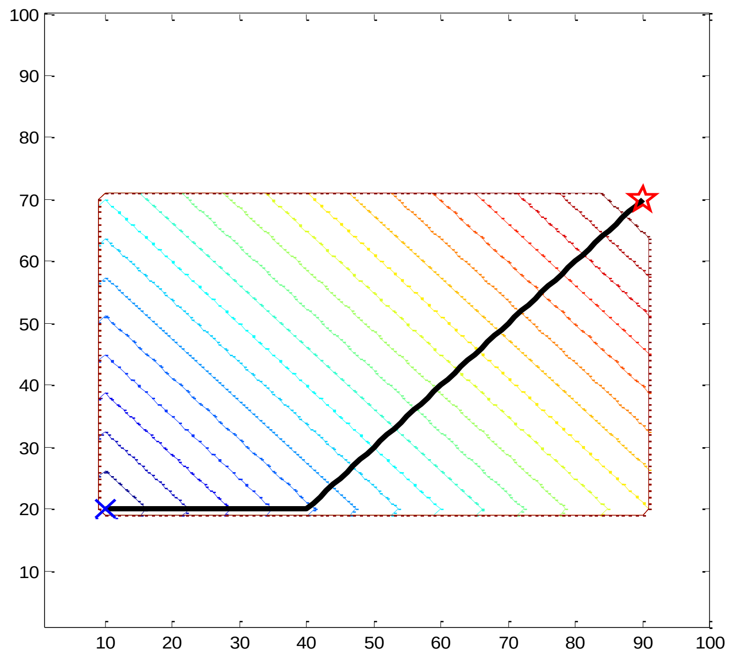
8-connexity Breadth-First



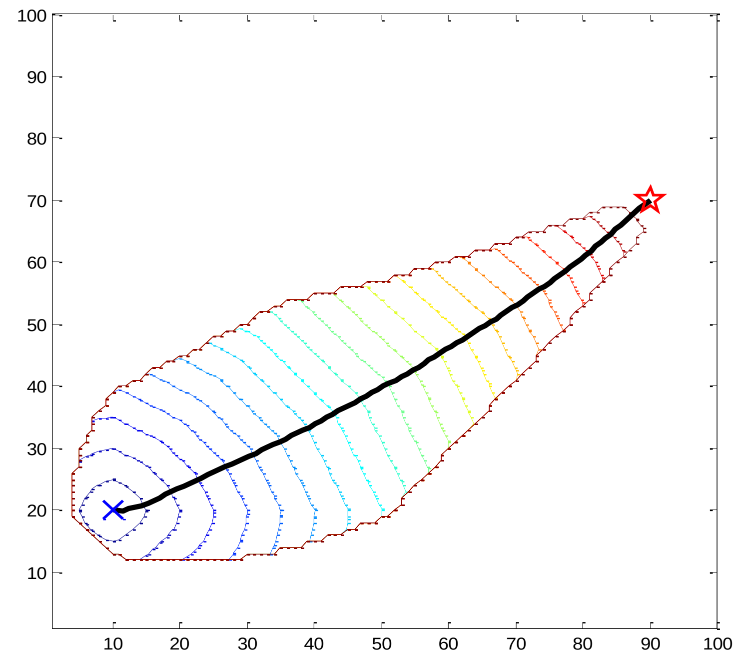
4-connexity Fast Marching (Sethian, 1996)

A* versus FM*

A* = Breadth-First + heuristic
FM* = Fast Marching + heuristic



4-connexity A* (Nilsson, 1968)

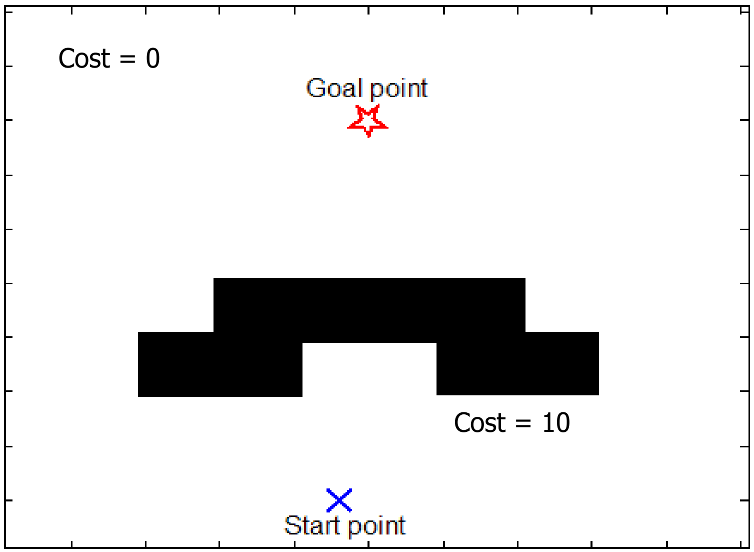


4-connexity FM*

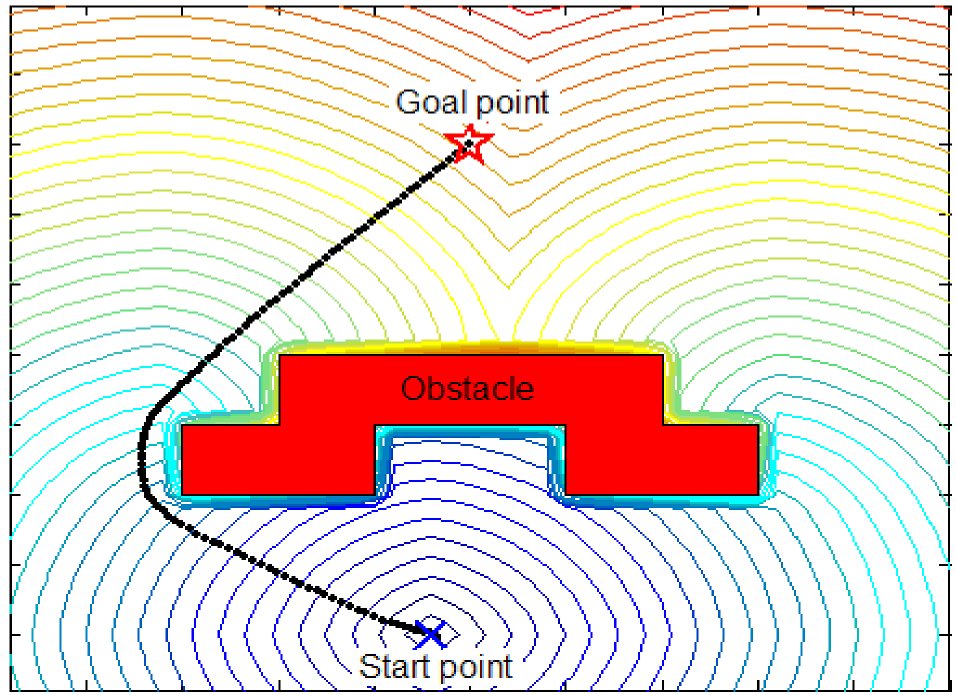
On a grid with N points: FM complexity = A* complexity = O(N.log(N)):

- Maximum cost of sorting the queue: log(N)
- Maximum number of iterations: N

Non-convex obstacles: Fast Marching method

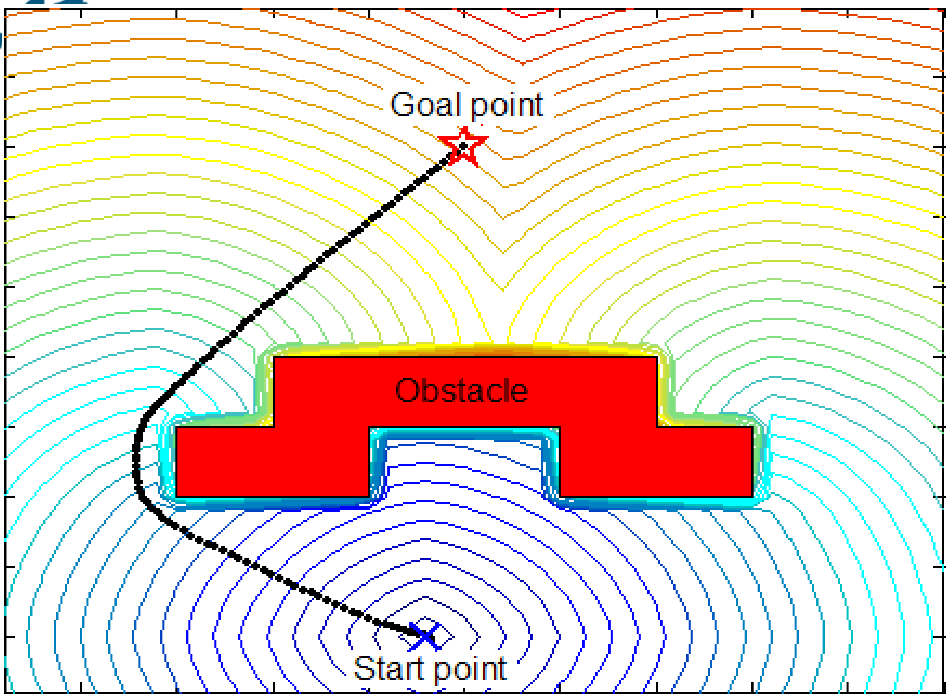


Cost map

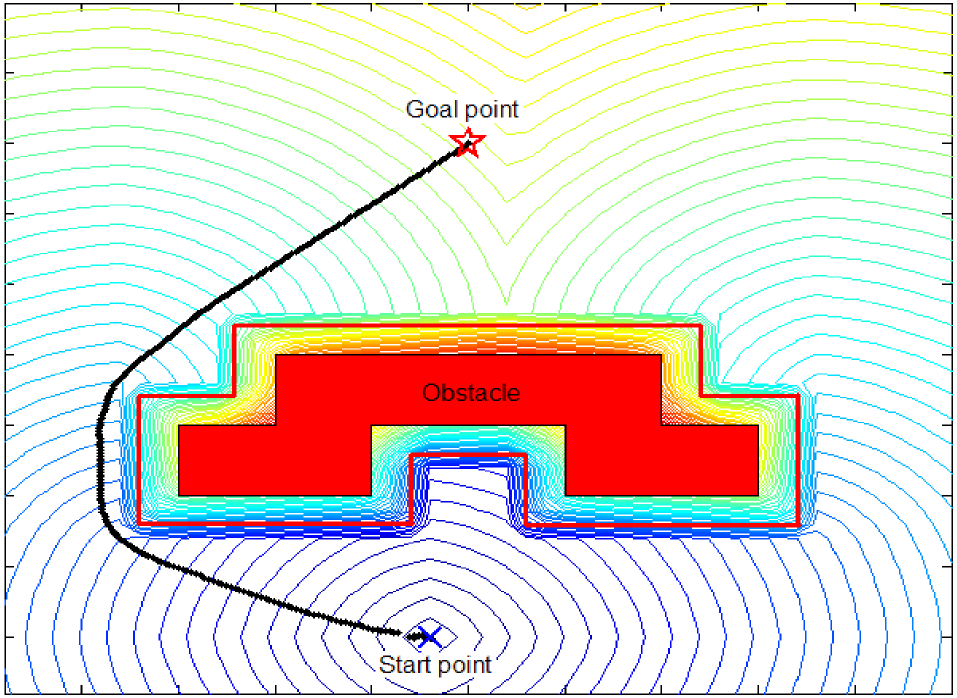


Distance map (Fast Marching) and optimal trajectory (gradient descent)

Non-convex obstacles: dilatation + Fast Marching



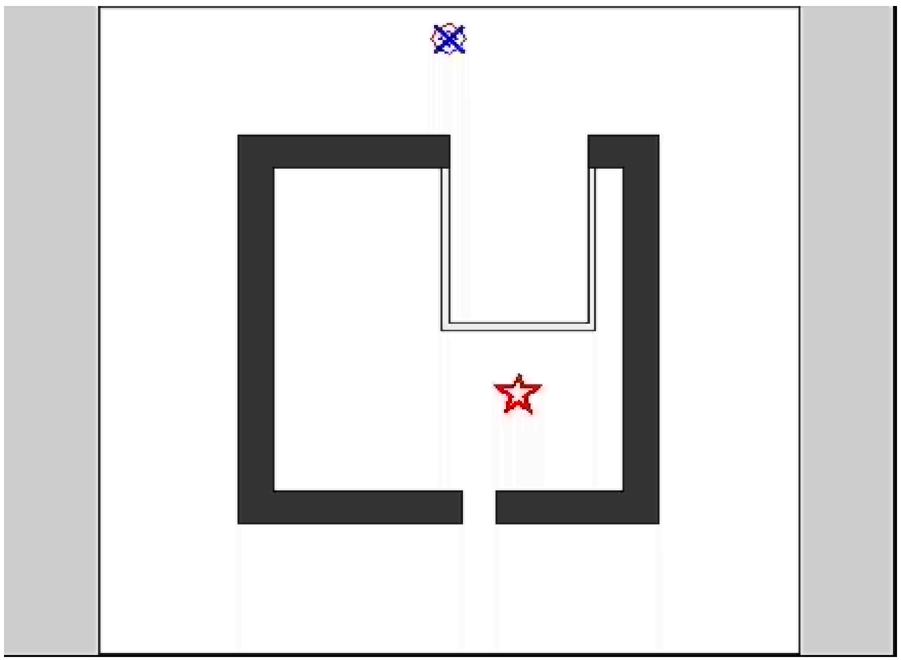
Trajectory avoiding obstacle



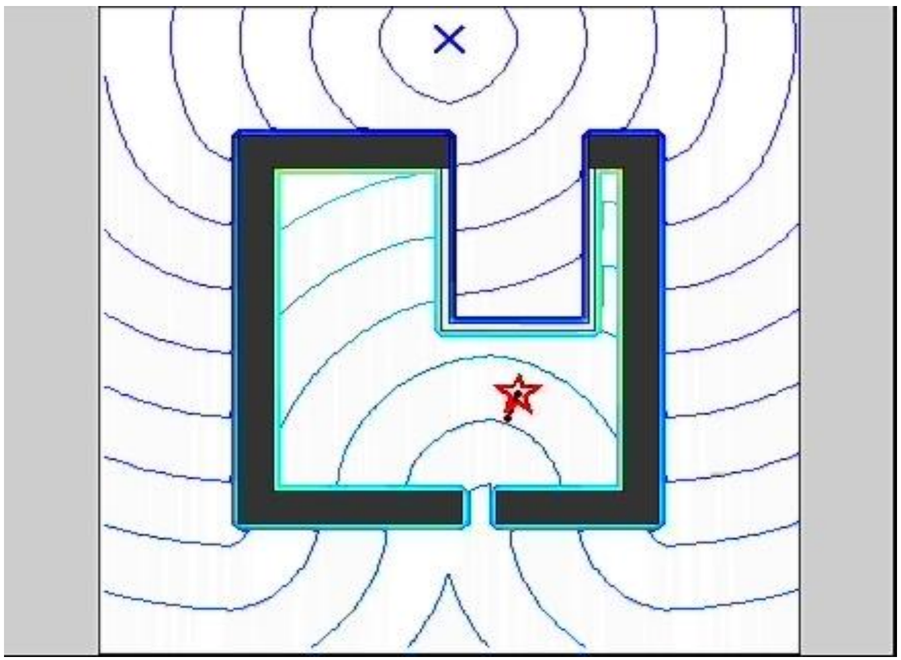
Safer trajectory avoiding dilated obstacle

Application: harbour obstructed by a net

Harbour 2D simulation

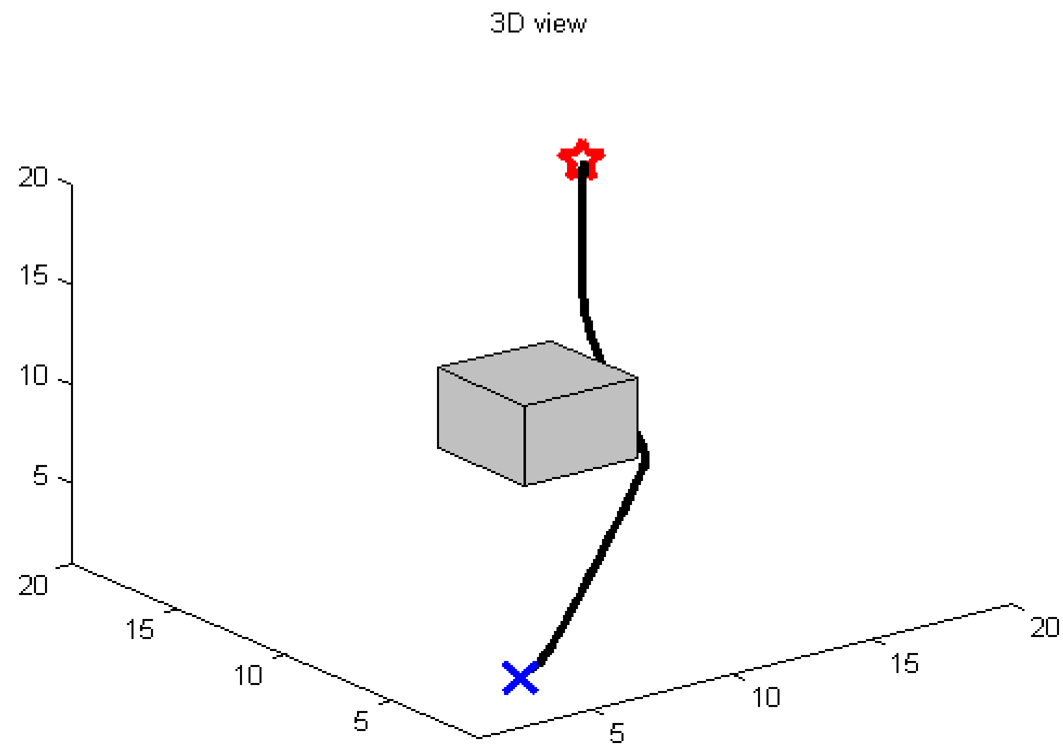


Distance map computation



Gradient descent computation

Application: 3D trajectory planning



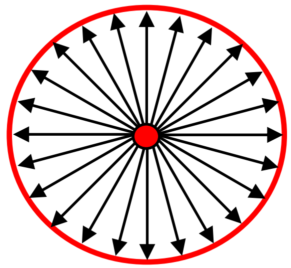
3D optimal trajectory using 3D Fast Marching algorithm

Presentation overview

Anisotropic Fast Marching: theory

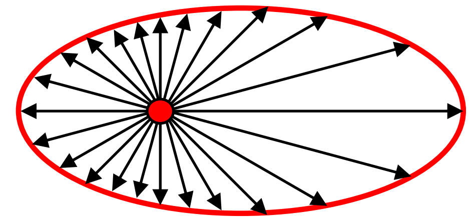
Eikonal equation

$$\|\nabla u(x)\| = \tau(x)$$



Hamilton-Jacobi equation

$$\|\nabla u(x)\| = \tilde{\tau}(x, \nabla u)$$

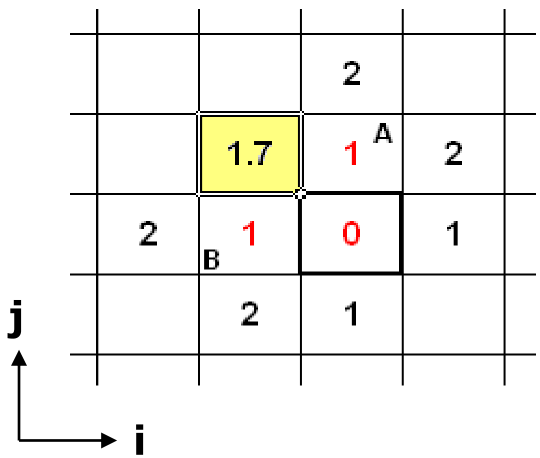


Anisotropic cost function

$$\tilde{\tau}(x, \nabla u) = \tau_O(x) + \tau_F(x, \nabla u)$$

Upwind scheme

$$\nabla u = \begin{pmatrix} -(u - u_A) \\ (u - u_B) \end{pmatrix}$$



$$\tau_F(x, \nabla u) = \alpha \left(1 - \frac{\nabla u(x).F(x)}{Q(x)} \right)$$

$$Q(x) = (\tau(x) + 2\alpha)\sup_\Omega(\|F\|) \text{ so that } \forall x \in \Omega, \left| \frac{\nabla u(x).F(x)}{Q(x)} \right| \leq 1$$

τ_F linear for u :

$$\tau_F(x, \nabla u) = \alpha \left(1 - \frac{-(u - u_A) F_i(x) + (u - u_B) F_j(x)}{Q(x)} \right)$$

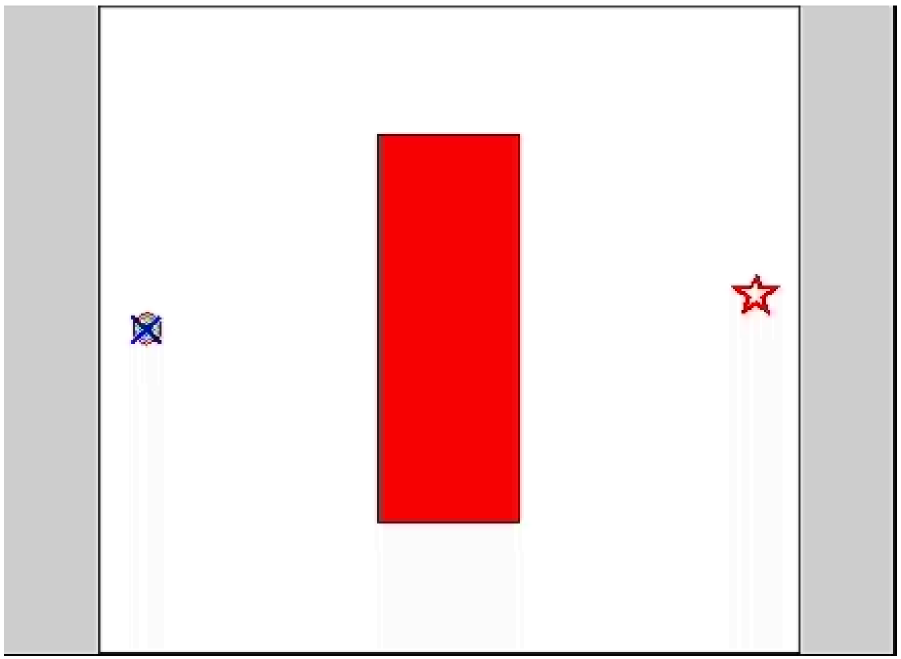
Quadratic equation for u

$$(u - u_A)^2 + (u - u_B)^2 = \tau^2$$

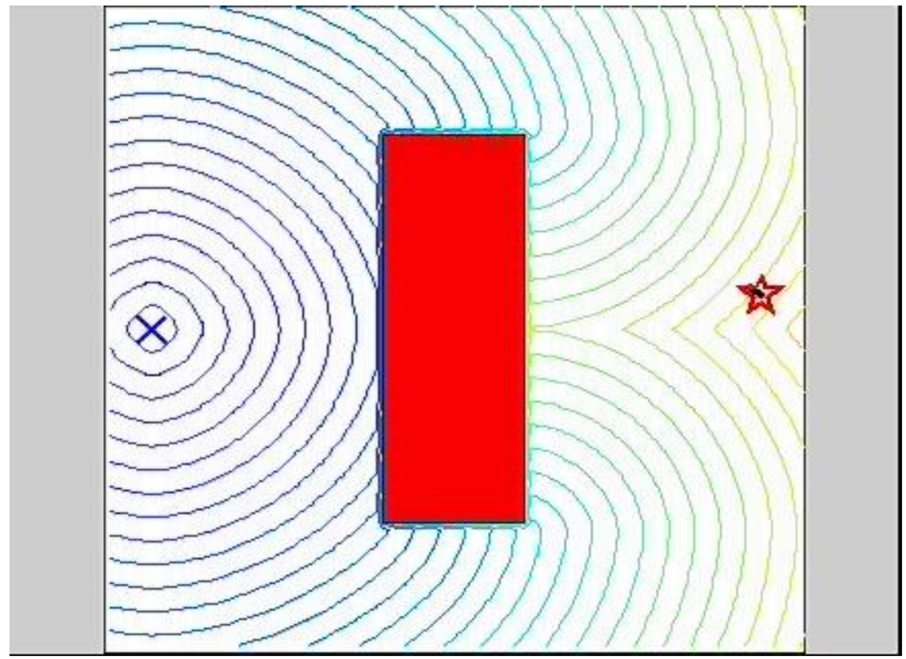
Quadratic equation for u

$$(u - u_A)^2 + (u - u_B)^2 = \tau_O{}^2 + \tau_F{}^2 + 2\tau_O\tau_F$$

Isotropic Fast Marching



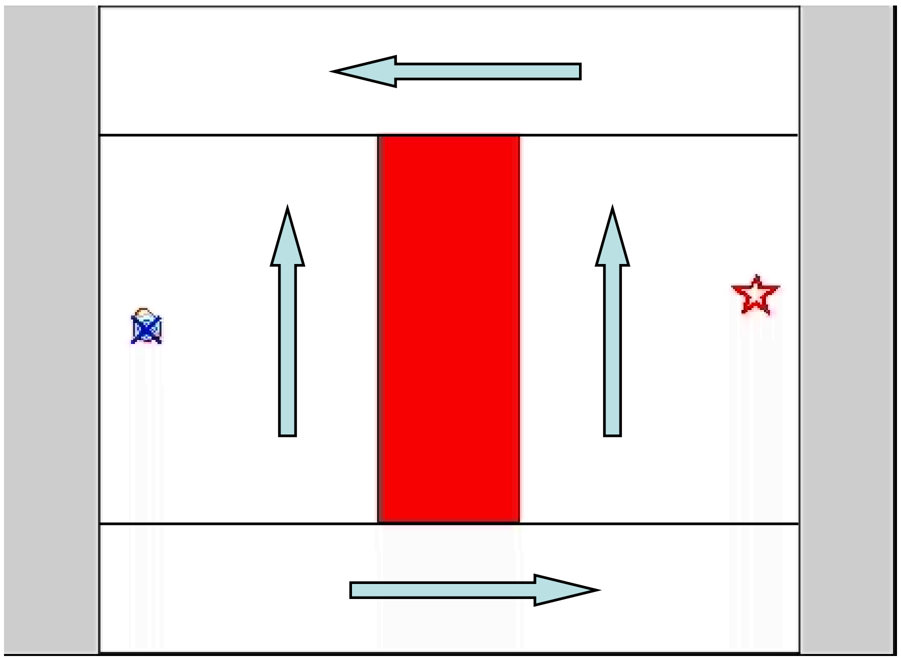
Distance map computation



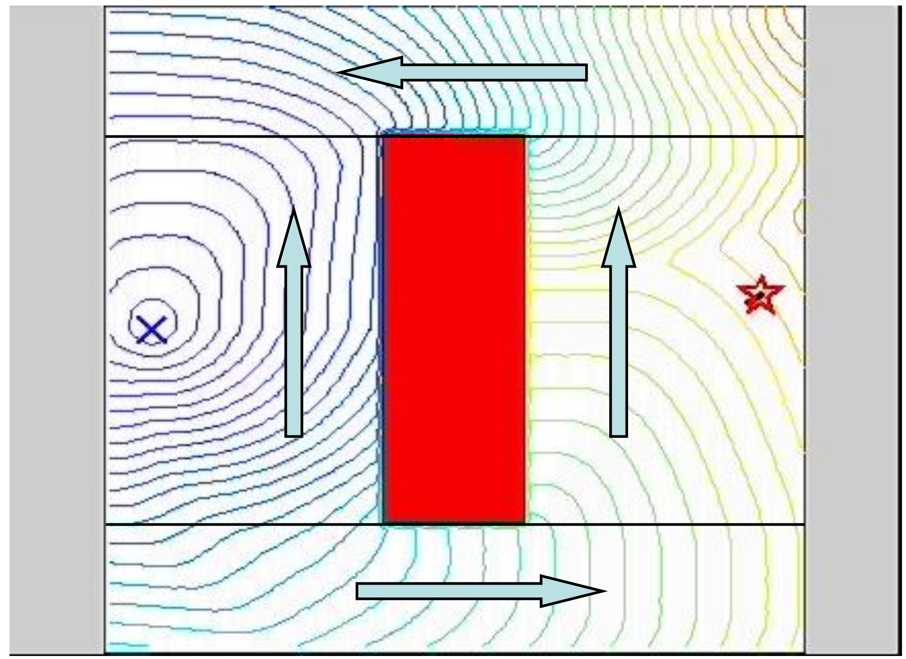
Gradient descent computation

Anisotropic Fast Marching: simulation

Anisotropic Fast Marching



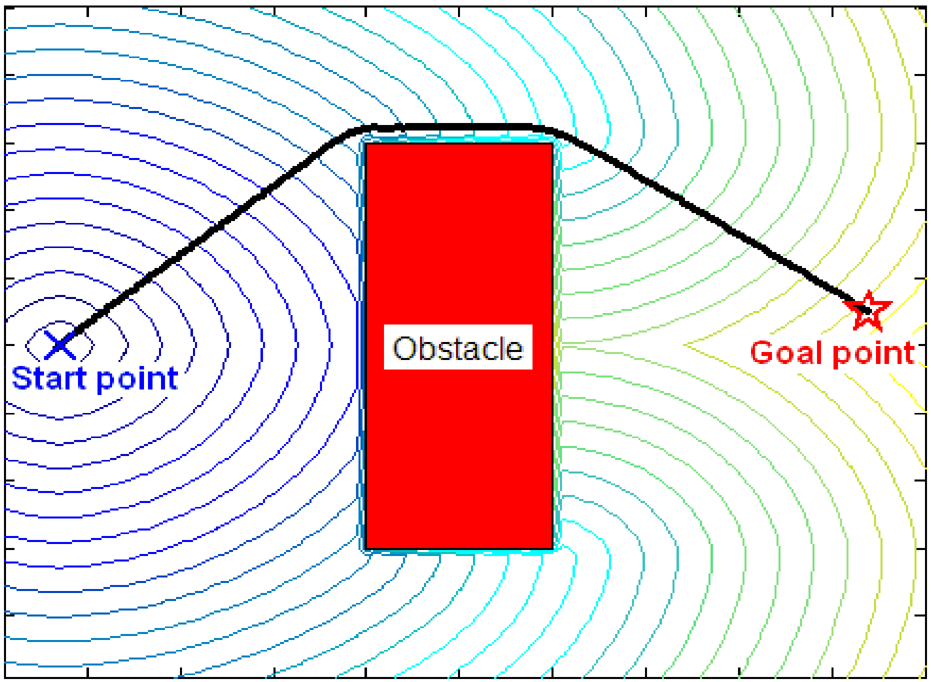
Distance map computation



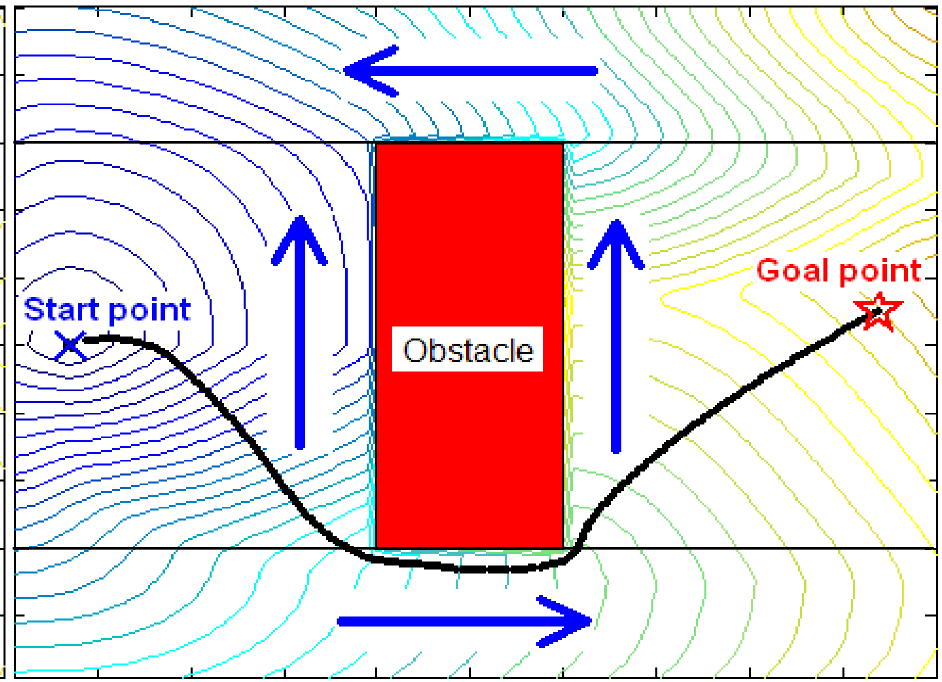
Gradient descent computation

Anisotropic Fast Marching: results

Isotropic Fast Marching



Anisotropic Fast Marching

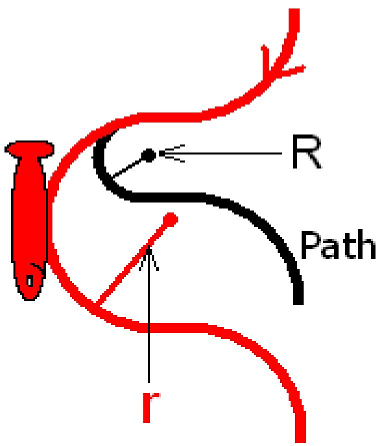


Gain = 10 %

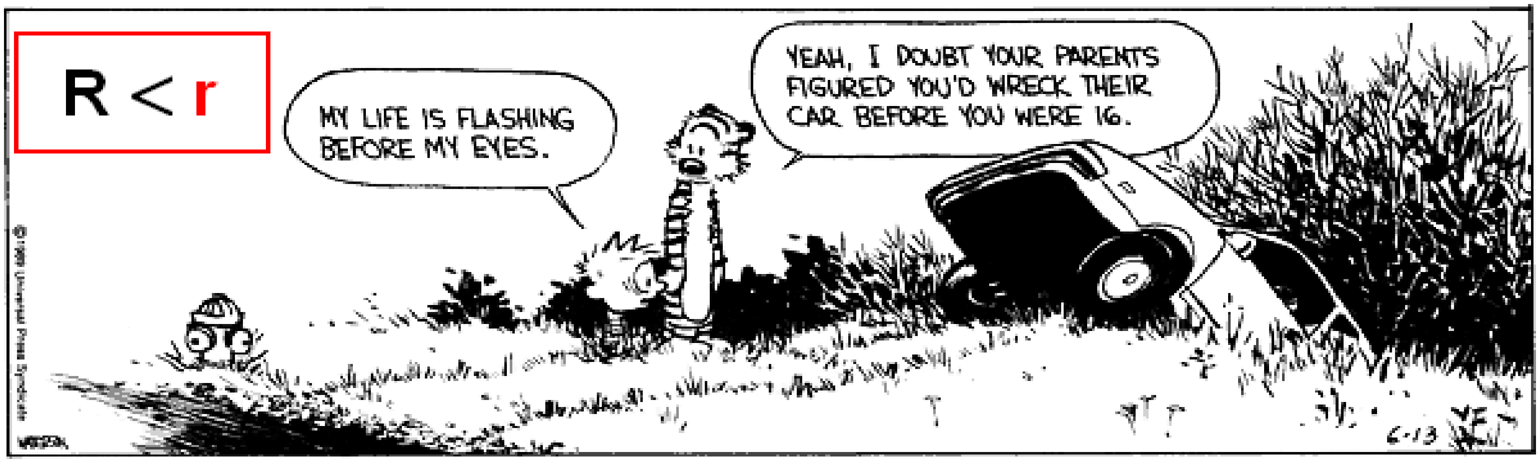
Presentation overview

R : trajectory curvature radius

r : turning radius of the vehicle



R < r



R > r

Trajectory planning under curvature constraints

Trajectory

$$C : \begin{array}{ccc} [0,1] & \rightarrow & \Omega \\ s & \mapsto & C(s) \end{array} \quad C(0) = x_{start} \quad C(1) = x_{end}$$

Metric (cost function τ)

$$\rho(x_1, x_2) = \int_{[0,1]} \tau\left(C_{x_1,x_2}(s)\right) ds$$

Functional minimization problem

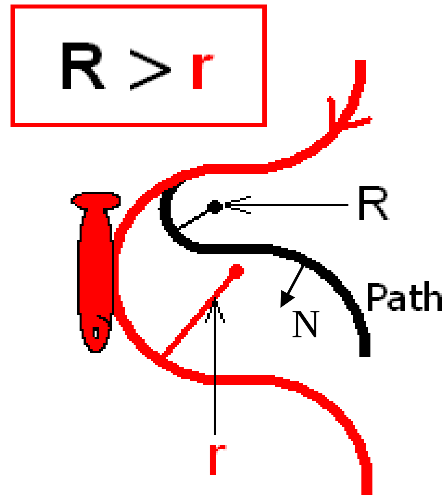
$$u(x_1, x_2) = \inf_{\{C_{x_1,x_2}\}} \rho(x_1, x_2)$$

Euler-Lagrange equation

$$\frac{\tau}{R} - \nabla\tau . N = 0$$

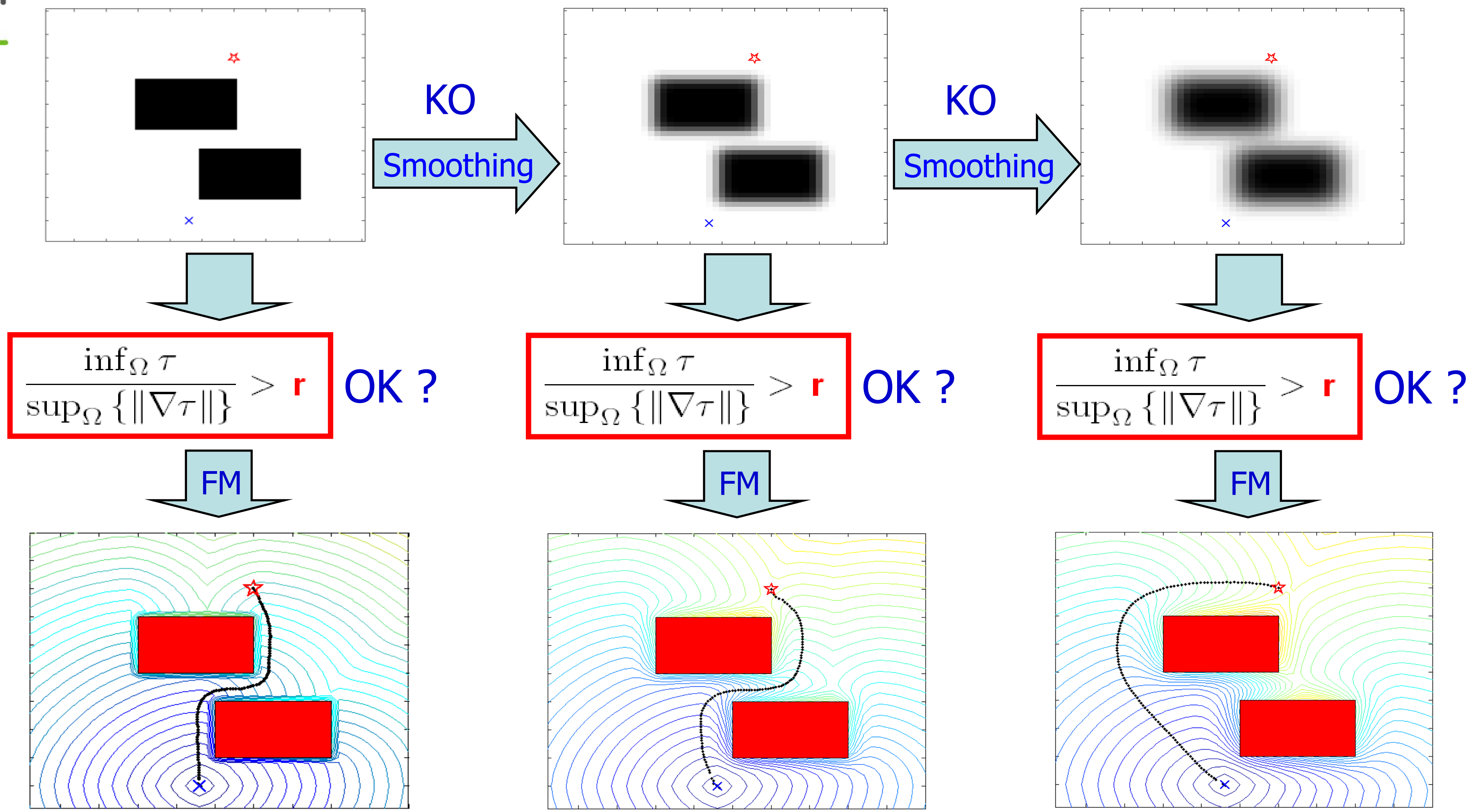
$$R \geq \frac{\inf_\Omega \tau}{\sup_\Omega \|\nabla\tau\|} > r$$

(Cohen and Kimmel, 1997)



Smoothing τ to decrease $\sup_\Omega \|\nabla\tau\|$

Trajectory planning under curvature constraints

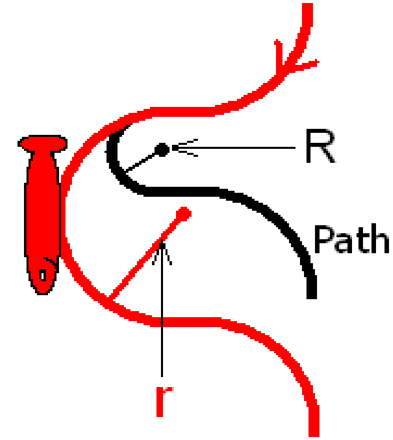


Lower bounds on the curvature radius

Cost function

$$\tilde{\tau}(x, \nabla u) = \tau_O(x) + \tau_F(x, \nabla u)$$

$$\tau_F(x, \nabla u) = \alpha\left(1 - \frac{\nabla u(x).F(x)}{Q(x)}\right)$$



Euler-Lagrange equation

$$\frac{\tau_O + \tau_F}{R} - \nabla\tau_O.N \pm \frac{\alpha}{Q}\left(\frac{\partial F_i}{\partial y} - \frac{\partial F_y}{\partial x}\right) = 0$$

(Petres and Pailhas, 2005)

$$R \geq \frac{\inf_\Omega \tau_O}{\sup_\Omega \|\nabla\tau_O\| + \frac{2\alpha}{\inf_\Omega Q}\|J_F\|_\infty} > r$$

Isotropic case

$$\frac{\tau}{R} - \nabla\tau.N = 0$$

$$R \geq \frac{\inf_\Omega \tau}{\sup_\Omega \|\nabla\tau\|} > r$$

- Smoothing τ_O to decrease $\sup_\Omega \|\nabla\tau_O\|$
- Smoothing the field of force F to decrease $\|J_F\|_\infty$

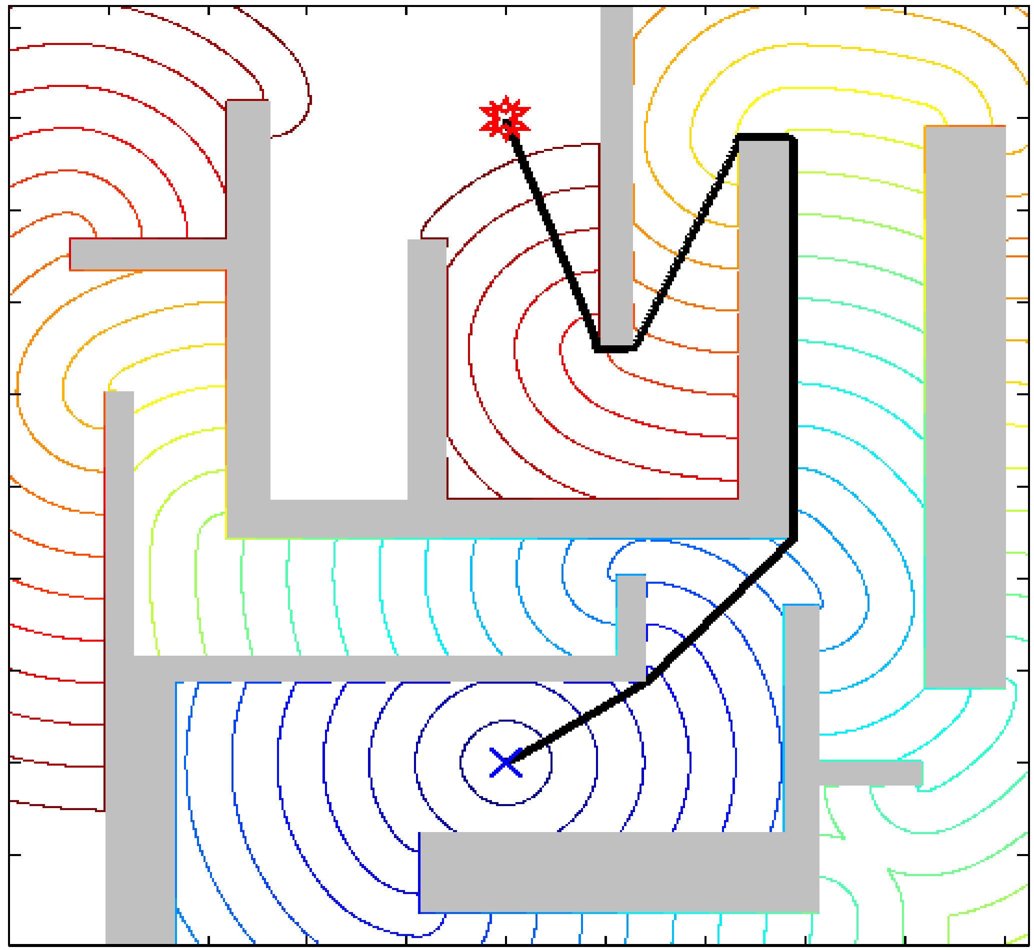
Presentation overview

Multiresolution FM based trajectory planning

Operations:

1) 1000x1000 grid of pixels

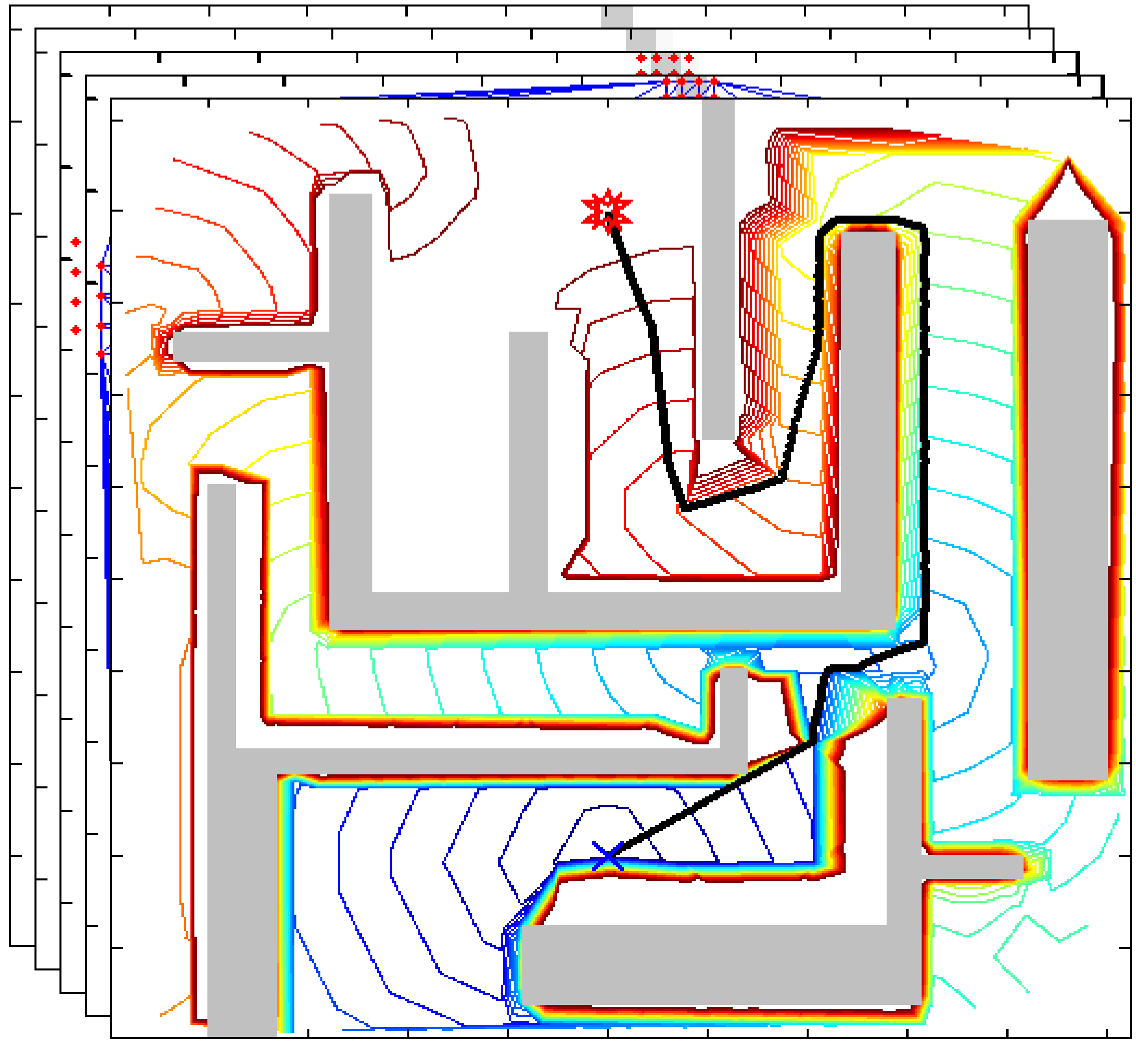
2) FM on the grid



Multiresolution FM based trajectory planning

Operations:

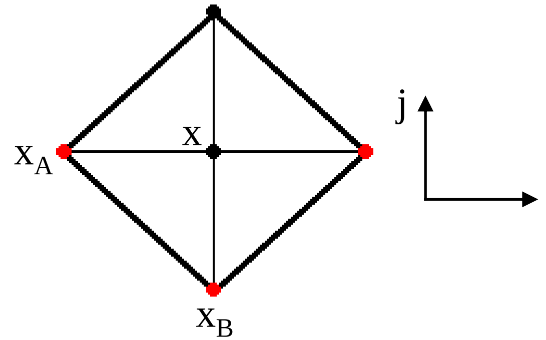
1) 1000x1000 grid of pixels
2) Quadtree decomposition
3) Mesh of 1400 vertices
4) FM on the mesh



Multiresolution FM based trajectory planning: upwind scheme

Cartesian grid

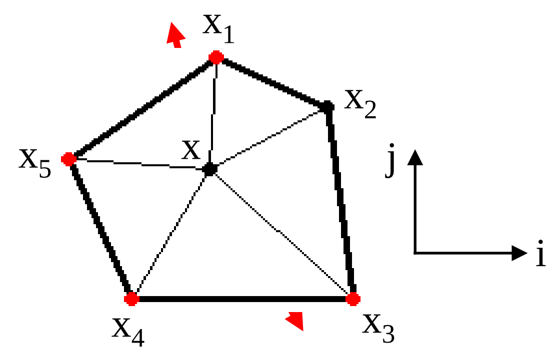
$$(u - u_A)^2 + (u - u_B)^2 = \tau^2$$



Unstructured mesh

$$(a^T Q a)u^2 + (2a^T Q b)u + b^T Q b = \tau^2$$

(Sethian and Vladimirsky, 2000)



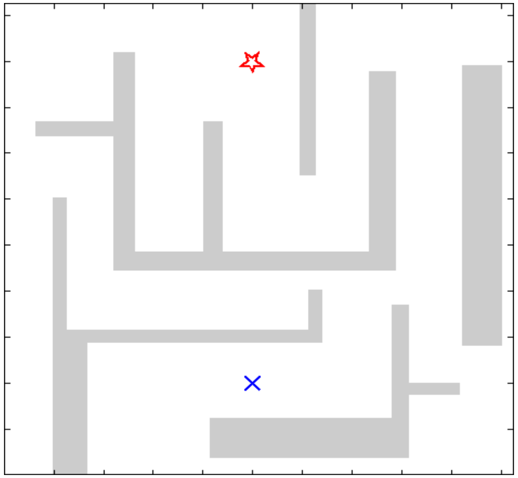
Notations:

$$T_i = \frac{x - x_i}{\|x - x_i\|} \qquad T = (T_1, T_2, \ldots, T_n) \qquad Q = (T^T T)^{-1}$$

$$a_i = \frac{1}{\|x - x_i\|} \qquad a = \begin{pmatrix} a_1 \\ a_2 \\ \ldots \\ a_n \end{pmatrix} \qquad b_i = -\frac{u_i}{\|x - x_i\|} \qquad b = \begin{pmatrix} b_1 \\ b_2 \\ \ldots \\ b_n \end{pmatrix}$$

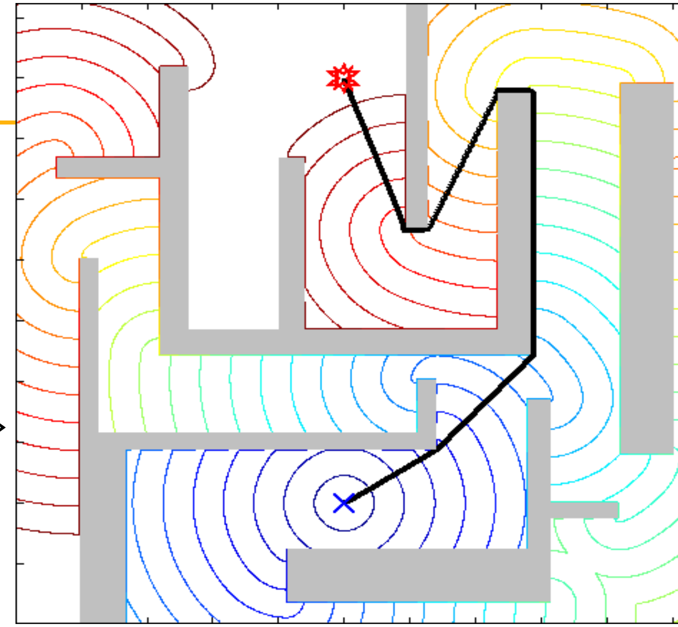
$$u = \min(u_1, u_2, u_3)$$

Recapitulative



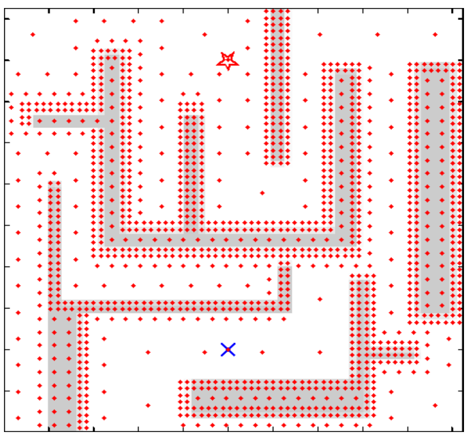
Original 1000x1000 image

100 seconds

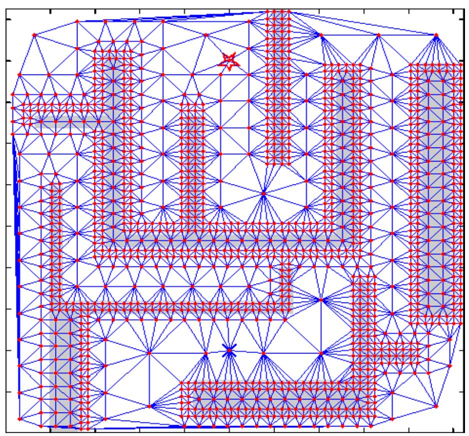


Trajectory using FM on the grid

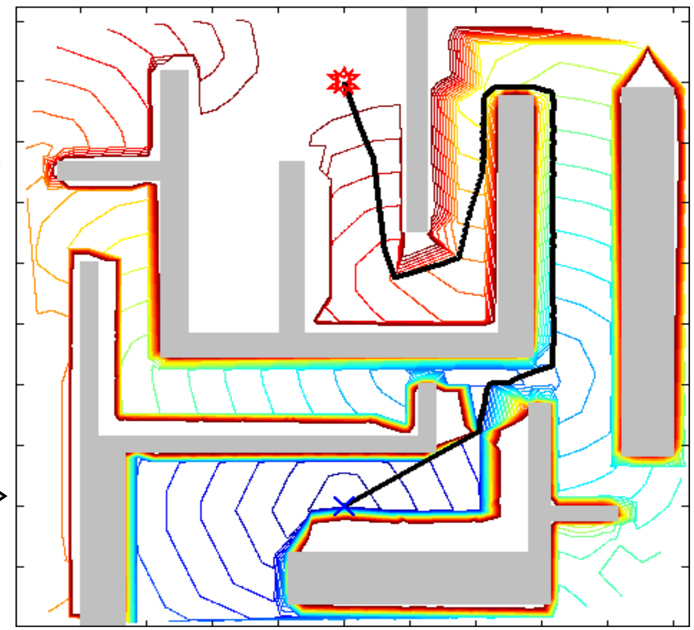
1 second



Quadtree decomposition



Mesh with 1400 vertices

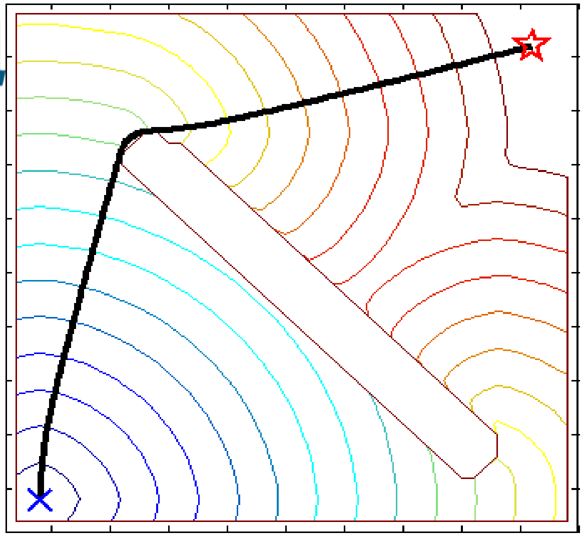


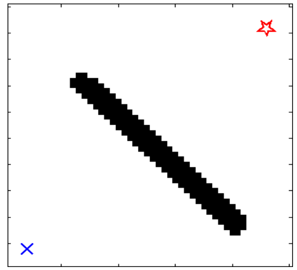
Trajectory using FM on the mesh

Presentation overview

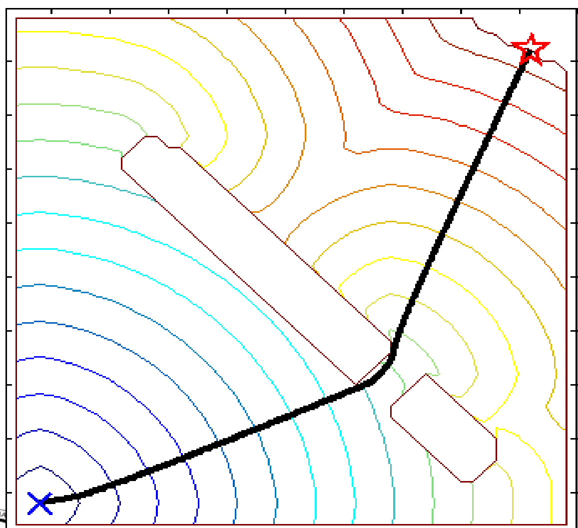
FM based trajectory planning in dynamic environments

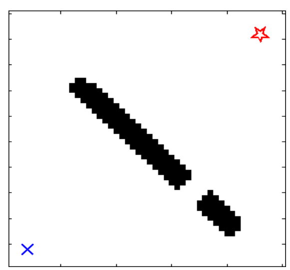
E* algorithm (Philippsen and Siegwart, 2005)

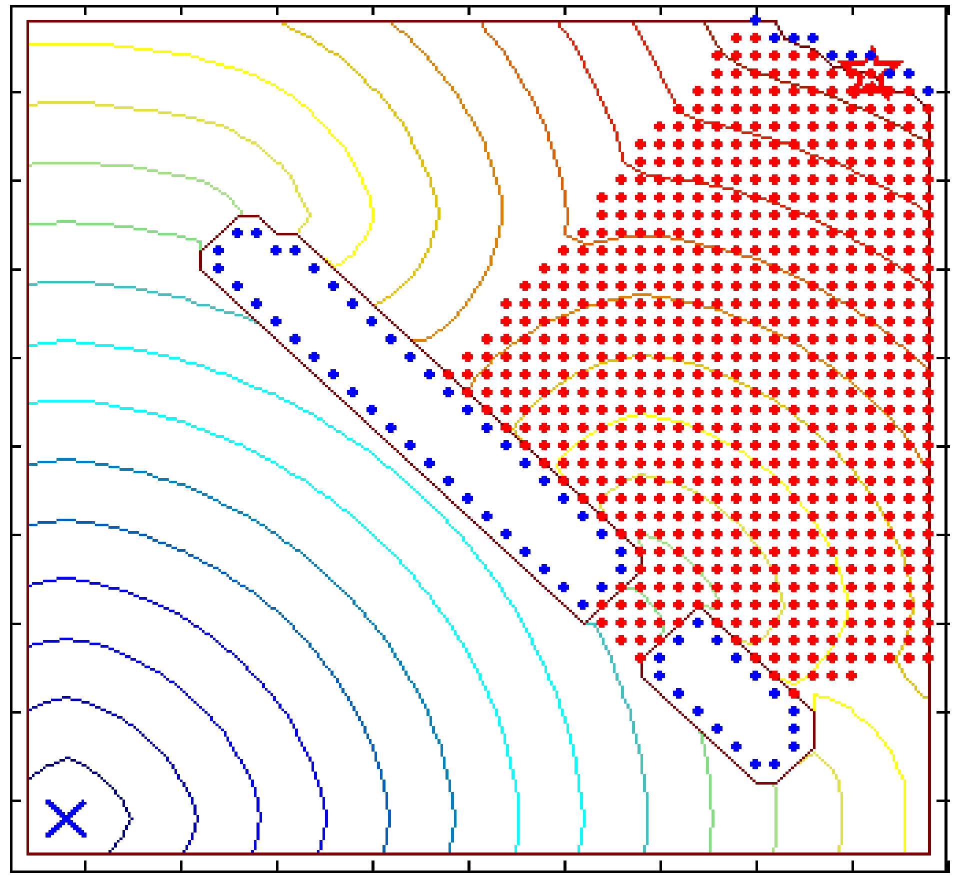




Cost map







Updated grid points

Dynamic replanning: video of trials in Scotland

Presentation overview

I. Introduction to FM based trajectory planning

II. Trajectory planning under directional constraints

III. Trajectory planning under curvature constraints

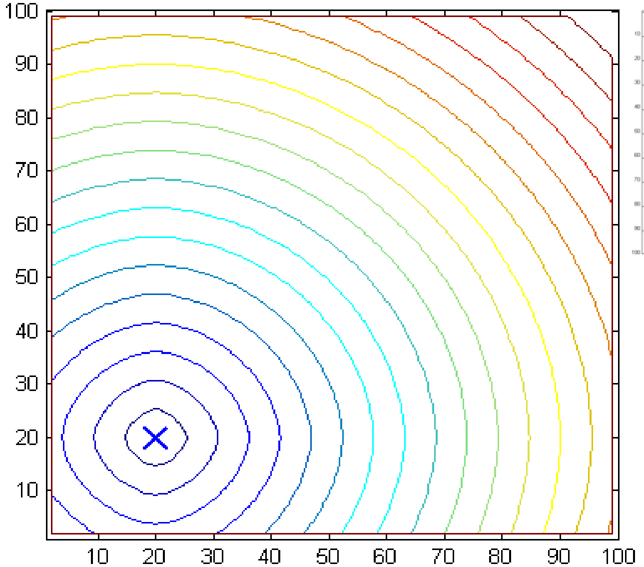
IV. Multiresolution FM based trajectory planning

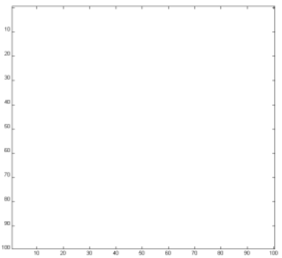
V. FM based trajectory planning in dynamic environments

VI. Trajectory planning under visibility constraints

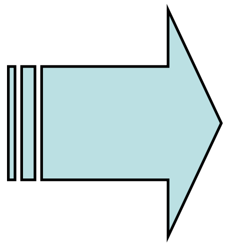
Trajectory planning under visibility constraints

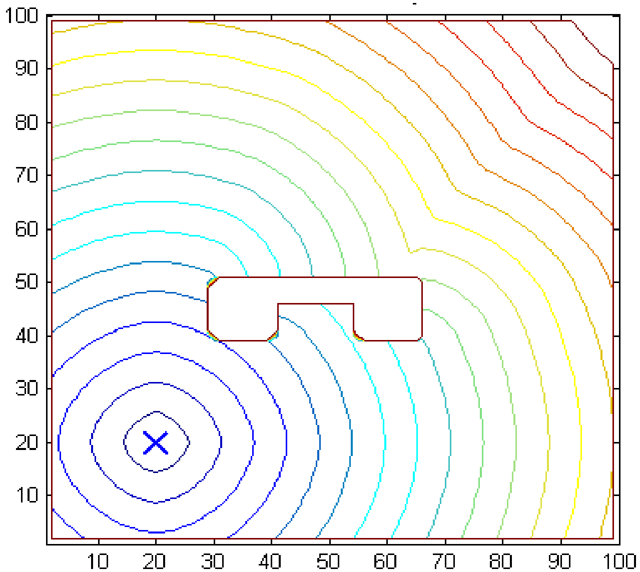
Visibility map: homogeneous media



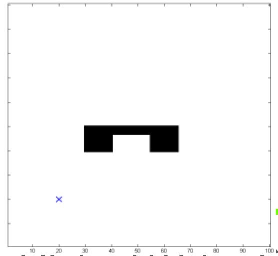


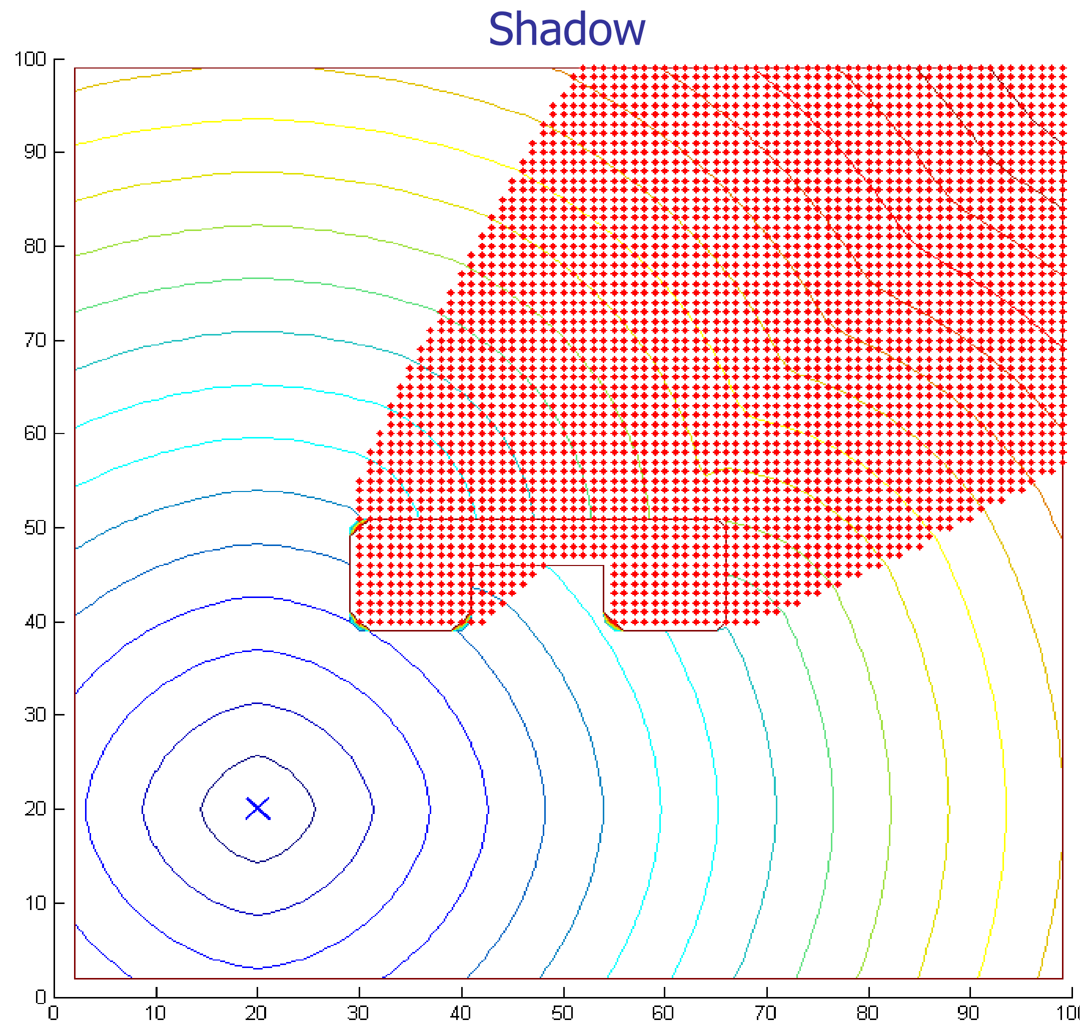
Cost map





Cost map

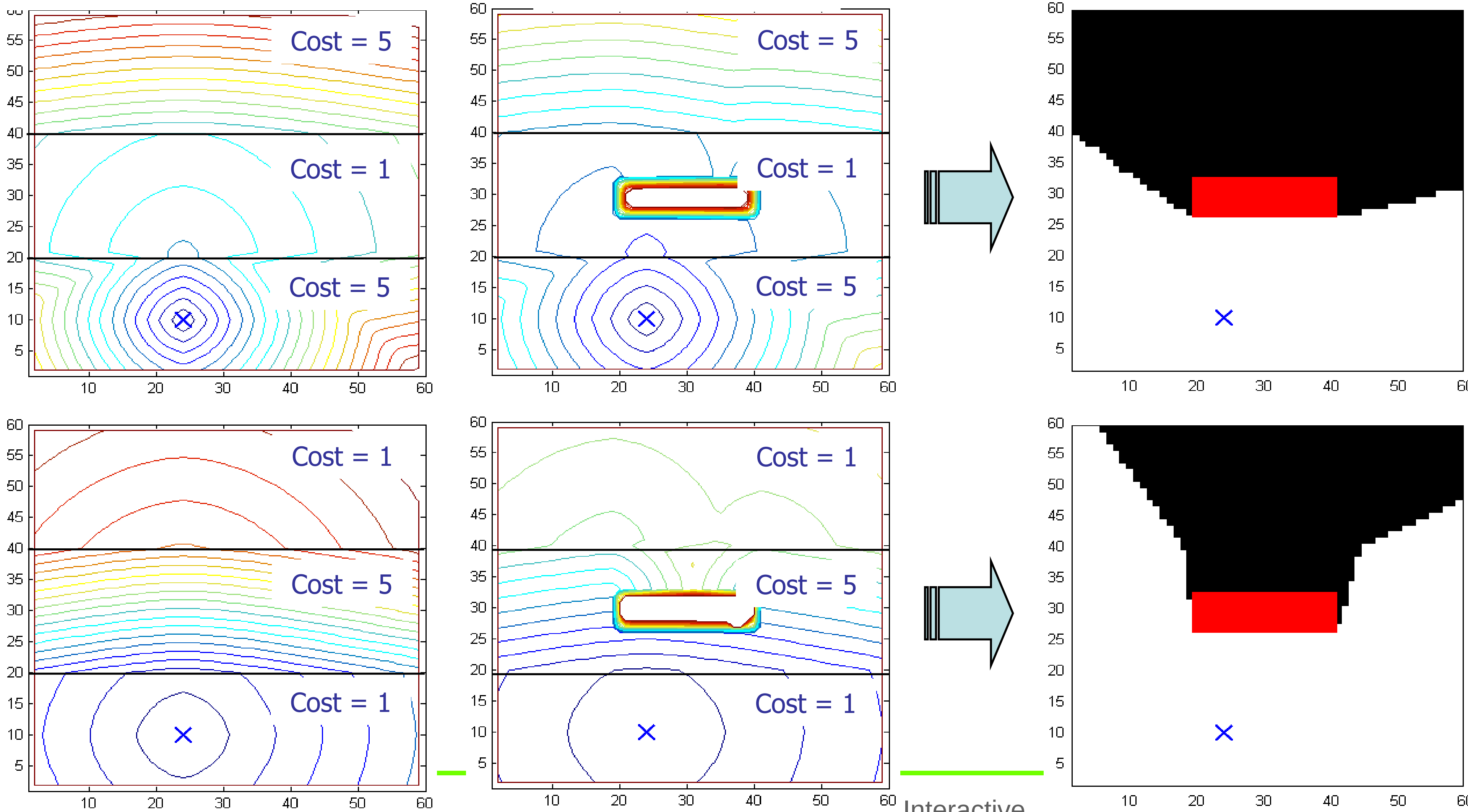




Trajectory planning under visibility constraints

Visibility map: hetrogeneous media

Without obstacle | With obstacle | Shadow

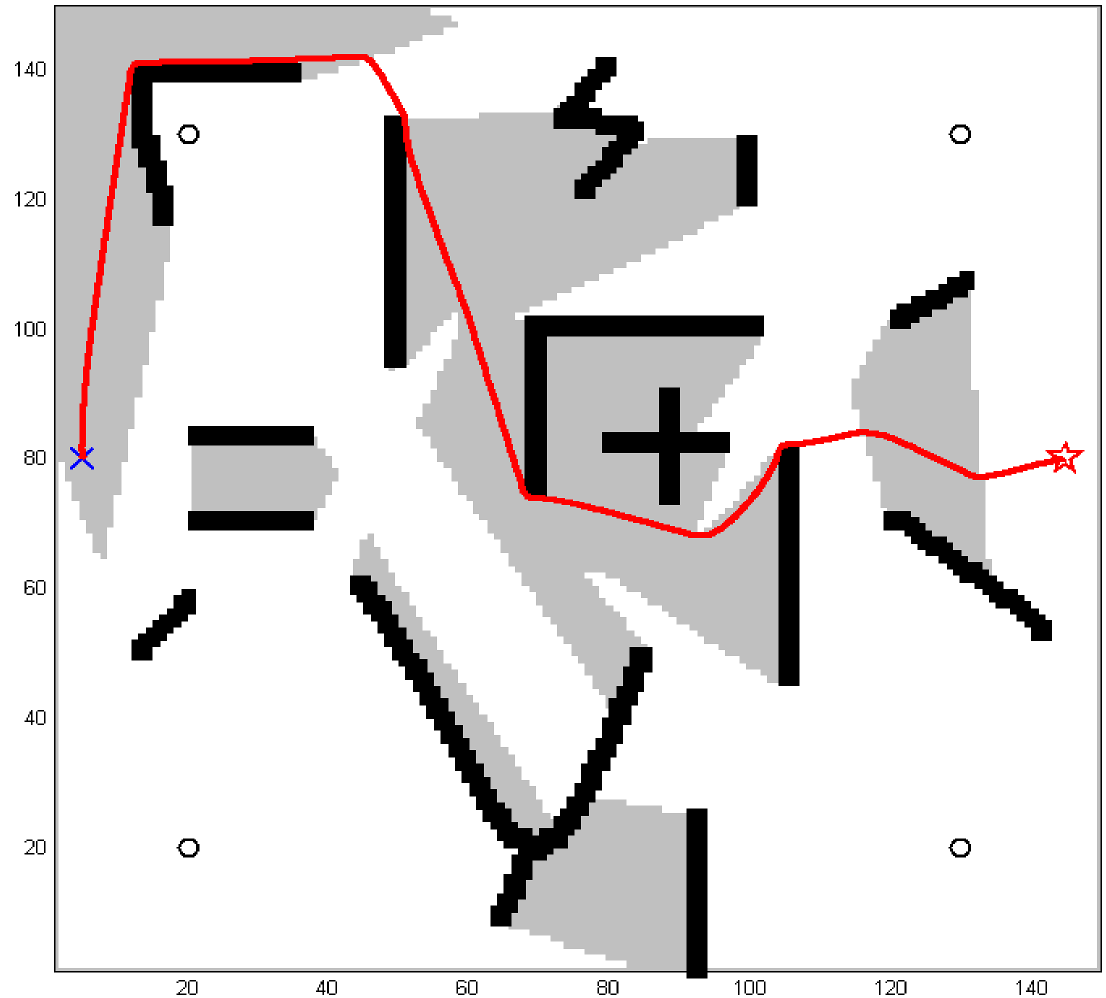


Trajectory planning under visibility constraints

Application in covert robotics

1 sentry in the 4 corners

Cost of covered areas : 1
Cost of exposed areas : 10
Cost of obstacles : 100



Conclusion

Recap of the talk

- FM* algorithm: FM + heuristic
- Directional constrained TP: anisotropic FM
- Curvature constrained TP: isotropic and anisotropic media
- Multiresolution TP: FM + adaptive mesh generation
- Dynamic replanning: E* algorithm, comparative study, in-water trials
- Visibility-based TP: E* based visibility map, heterogeneous environments

Main advantages of Fast Marching methods applied to trajectory planning

- Accuracy, robustness → reliability
- Curvature constraints → underactuated AUV
- Fields of force → new domains of applicability
- Simplicity → easy integration on real systems

References on chapter 1

- E. W. Dijkstra, « A Note on Two Problems in Connexion with Graphs », *Numerische Mathematik*, vol. 1, pp. 269–271, 1959.

- P. E. Hart, N. J. Nilsson, B. Raphael, « A Formal Basis for the Heuristic Determination of Minimum Cost Paths », *IEEE Transactions on Systems Science and Cybernetics*, vol. 4(2), pp. 100–107, 1968.

- J. A. Sethian, « Level Set Methods and Fast Marching Methods », *Cambridge University Press*, 1999.

References on chapter 2

- A. Vladimirsky, Ordered Upwind Methods for Static Hamilton-Jacobi Equations: Theory and Algorithms, *SIAM Journal on Numerical Analyzis*, vol. 41(1), pp. 325-363, 2003.

- M. Soulignac, P. Taillibert, M. Rueher, « Adapting the Wavefront Expansion in Presence of Strong Currents », *IEEE International Conference on Robotics and Automation*, pp. 1352-1358, 2008.

- B. Garau, A. Alvarez, G. Oliver, « Path Planning of AUV in Current Fields with Complex Spatial Variability: an A* approach », *IEEE International Conference on Robotics and Automation*, pp. 194-198, 2005.

References on chapter 3

- L. D. Cohen, Ron Kimmel, « Global Minimum for Active Contour Models: A Minimal Path Approach », *International Journal on Computer Vision*, vol. 24(1), pp. 57-78, 1997.
- S. M. Lavalle, « Planning Algorithms », *Cambridge University Press*, 2006.

References on chapter 4

- J. A. Sethian, A. Vladimirsky, « Fast Methods for the Eikonal and Related Hamilton-Jacobi Equations on Unstructured Meshes », *Applied Mathematics*, vol. 97(11), pp. 5699-5703, 2000.

- D. Ferguson, A. Stentz, « Multi-Resolution Field D* », *International Conference on Intelligent Autonomous Systems*, 2006.

- A. Yahja, A. Stentz, S. Singh, B. Brumitt, « Framed-Quadtree Path Planning for Mobile Robots Operating in Sparse Environments », *IEEE International Conference on Robotics and Automation*, vol. 1, pp. 650-655, 1998.

References on chapter 5

- R. Philippsen, R. Siegwart, « An Interpolated Dynamic Navigation Function », *IEEE International Conference on Robotics and Automation*, pp. 3782-3789, 2005.

- D. Ferguson, A. Stentz, « Using Interpolation to Improve Path Planning: The Field D* Algorithm », *Journal of Field Robotics*, vol. 23(2), pp. 79-101, 2006.

- A. Stentz, « Optimal and Efficient Path Planning for Partially-Known Environment », *IEEE International Conference on Robotics and Automation*, pp. 3310-3317, 1994.

References on chapter 6

- J. A. Sethian, « Level Set Methods and Fast Marching Methods », *Cambridge University Press*, 1999.

- Y.-H. Tsai, L.-T. Cheng, S. Osher, P. Burchard, G. Sapiro, « Visibility and Its Dynamics in a PDE Based Implicit Framework », *Journal of Computational Physics*, vol. 199(1), pp. 260-290, 2004.

Main sources

- C. Pêtrès, Y. Pailhas, P. Patron, Y.Petillot, J. Evans, D. Lane, « Path Planning for Autonomous Underwater Vehicles », *IEEE Transactions on Robotics*, vol. 23(2), pp. 331-341, 2007.

- C. Pêtrès, « Trajectory Planning for Autonomous Underwater Vehicles », *Ph.D. Thesis*, Heriot-Watt University, Ocean Systems Laboratory, Edinburgh, Scotland, 2007.

Webpage: http://clement.petres.googlepages.com/